Math 251 - Differential Calculus

Core Differentiation Rules

Power Rule

$$\frac{d}{dx}x^n = n \cdot x^{n-1}$$

Product Rule

$$\frac{d}{dx}f(x) \cdot g(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Quotient Rule

$$\frac{d}{dx}\frac{f(x)}{g(x)} = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

Chain Rule

$$\frac{d}{dx}f(g(x)) = f'(g(x)) \cdot g'(x)$$

Limits

Limit exists if: $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x)$

Properties of Limits

1. $\lim_{x \to a}[cf(x)] = c\lim_{x \to a} f(x)$
2. $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$
3. $\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$
4. $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$ provided $\lim_{x \to a} g(x) \neq 0$
5. $\lim_{x \to a}[f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n$
6. $\lim_{x \to a}\left[\sqrt[n]{f(x)}\right] = \sqrt[n]{\lim_{x \to a} f(x)}$

Common Limits

$\lim_{x \to \infty} \mathbf{e}^x = \infty$ & $\lim_{x \to -\infty} \mathbf{e}^x = 0$

$\lim_{x \to \infty} \ln(x) = \infty$ & $\lim_{x \to 0^+} \ln(x) = -\infty$

If $r > 0$ then $\lim_{x \to \infty} \frac{b}{x^r} = 0$

If $r > 0$ and x^r is real for negative x then $\lim_{x \to -\infty} \frac{b}{x^r} = 0$

n even : $\lim_{x \to \pm\infty} x^n = \infty$

n odd : $\lim_{x \to \infty} x^n = \infty$ & $\lim_{x \to -\infty} x^n = -\infty$

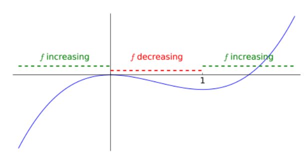
Trends and Features

Critical Points: *Occur wherever* $f'(c) = 0$, *or* $f'(c) = DNE$

Max: $f'(x)$ *goes from* $(-) \to (+)$ **Min:** $f'(x)$ *goes from* $(+) \to (-)$

Increasing: *Whenever* $f'(x) > 0$

Decreasing: *Whenever* $f'(x) < 0$



Concavity: *Concave up when* $f''(x) > 0$; *Concave down when* $f''(x) < 0$

Inflection points: *Occur whenever the concavity changes*

Common Derivatives

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(a^x) = a^x \ln(a)$$

$$\frac{d}{dx}(\mathbf{e}^x) = \mathbf{e}^x$$

$$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}, \quad x \neq 0$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x \ln a}, \quad x > 0$$

$$\frac{d}{dx}(\mathbf{e}^{f(x)}) = f'(x)\mathbf{e}^{f(x)}$$

$$\frac{d}{dx}(\sec[f(x)]) = f'(x)\sec[f(x)]\tan[f(x)]$$

↓

$sin(x)$
$cos(x)$
$-sin(x)$
$-cos(x)$
$sin(x)$

Trigonometry

$$sin^2\Theta + cos^2\Theta = 1$$

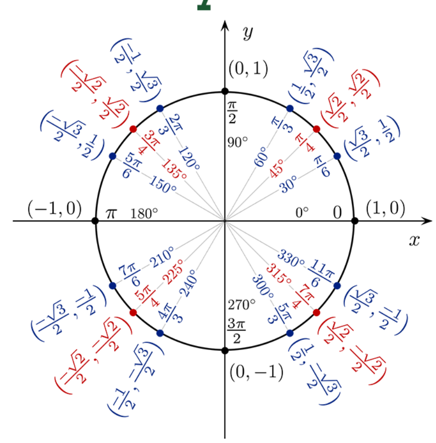
$$tan^2\Theta + 1 = sec^2\Theta$$

$$cos^2\Theta = \frac{1}{2} + \frac{cos(2\Theta)}{2}$$

$$sin^2\Theta = \frac{1}{2} - \frac{cos(2\Theta)}{2}$$

$$tan\Theta = \frac{sin\Theta}{cos\Theta}$$

$$cot\Theta = \frac{cos\Theta}{sin\Theta}$$



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